Can String Theory Avoid Cosmological Singularities?

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ABSTRACT. We consider the effective action for strings and describe in detail the evolution of a four dimensional homogeneous isotropic universe with matter included. We find that the evolution, which is singular in general, becomes singularity free if during certain Phase of the evolution, when the scale factor increases and the effective string coupling becomes strong, the universe is dominated by solitonic p-branes, $p = 0$ and/or -1, or by 'matter' for which $(pressure) \leq -\frac{1}{\sqrt{3}} \, (density)$. The mechanism in the case of branes is reminiscent of the recently discovered field theory mechanism where heavy states become light and resolve the moduli space singularities.

1. In cosmology the evolution of the universe generically has a singularity in the past, usually of big bang type as in standard cosmology. The energy scale involved is Planckian and quantum gravity effects are expected to play an important role in resolving the singularities. String theory is a leading candidate for a theory of quantum gravity and, more generally, of Physics at Planck scale. Therefore, it is natural to look for a resolution of the cosmological singularity within the context of string theory.

There have been numerous attempts to resolve the singularity within string theory. In [1]-[3] stringy T-duality symmetry is used to relate small size to large size where winding modes prevent the expansion of the universe to ∞ and thus, by T-duality, its collapse to zero size. Veneziano and collaborators have used this symmetry and developed the 'pre big bang cosmology' which contains a superinflationary branch and a standard cosmology branch [4, 5]. A 'graceful exit' from the first to the second branch will then result in a singularity free evolution of the universe. So far, however, the graceful exit has been problematic [5] (see [6] for a recent attempt to solve this problem).

Cosmological solutions to one-loop corrected string effective action have been studied in [7, 8] and it is shown [8] that non singular solutions exist when the spatial curvature is positive. Non singular solutions are also found upon including higher derivative terms that incorporate 'limiting curvature hypothesis' [9].

Cosmological solutions have been studied in M-theory [10] with non trivial Ramond-Ramond sector fields present. Decomposing the space into a set of maximally symmetric subspaces, a class of singularity free solutions have been obtained when the spatial curvature is positive [11]. These solutions are related to black p-brane solutions in a regime where space and time reverse their roles [11, 12]. Using such a role reversal of space and time, a scenario has been proposed recently [13] where big bang/crunch singularities are resolved at non singular horizons of higher dimensional quasi black hole solutions or, plausibly, at Dirichlet brane bound states [14] having no conventional space time interpretation.

In this letter, we consider the effective action for strings and analyse the evolution of a four dimensional homogeneous isotropic universe with matter included. The matter is taken to be a perfect fluid with density $\rho(> 0)$, pressure p, and equation of state $p = \gamma\rho$, $-1 \leq \gamma \leq 1$. We present a general analysis, applicable even in the absence of explicit solutions, from which the qualitative features of the evolution can be obtained. We find

that the evolution, which is singular in general, becomes singularity free if during certain Phase of the evolution, when the scale factor increases and the effective string coupling becomes strong, the universe is dominated by solitonic p-branes, $p = 0$ and/or -1, [1] or by 'matter' for which $\gamma \leq -\frac{1}{\sqrt{3}}$. Further evolution is described in detail.

It appears that either, or both, of these possibilities can be realised in string theory. The solitonic p-branes are heavy when the coupling is weak, but become light when the coupling is strong [14, 15]. Thus, they are likely to be produced copiously and dominate the universe during the relevent Phase. Using a result of Duff et al [16], it then follows that the solitonic p-branes, $p = 0$ and/or -1 do indeed avoid the singularity. This mechanism is reminiscent of the recently discovered field theory mechanism where heavy states - monopoles [17], Ramond-Ramond black holes [18], Dirichlet instantons [19] - become light and resolve the moduli space singularities.

Alternately, when the scale factor increases, as during the relevent Phase, a gas of p-branes may have negative pressure. Indeed $\gamma = -\frac{1}{3}$ for a gas of strings [1]-[4]. However, γ is not known for arbitrary p, but it is plausible that $\gamma \leq -\frac{1}{\sqrt{3}}$ for some p (suggested to us by G. Veneziano). If true then the singularity can be avoided by such 'matter' and the evolution becomes singularity free.

The paper is organised as follows. We first present the action and the equations of motion with matter included. We rewrite the equations in a form suitable for our analysis. We then analyse the evolution in detail and obtain the conditions for the evolution to be completely singularity free. We then consider how a gas of solitonic p-branes may avoid the singularity and conclude with a few remarks.

2. Consider the string effective action for graviton and dilaton in the following form:

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} \left(\chi R - \frac{\omega}{\chi} (\nabla \chi)^2 \right) \tag{1}$$

where $g_{\mu\nu}$ is the string σ-model metric, χ (≥ 0) is the dilaton, and Newton's constant is set equal to $\frac{1}{8\pi}$. The effective string coupling g_s is given by $g_s = \frac{1}{\sqrt{\chi}}$ (the square roots are to be taken with a positive sign always). ω

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[1] (-1)-branes are instantons but are also referred to as solitons here for the sake of brevity.

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is a constant equal to -1 for string theory but since the values of ω in the range $-\frac{3}{2} \leq \omega < -1$ will also be relevent in the following, we retain ω in (1) and assume only that $-\frac{3}{2} \leq \omega \leq -1$. Action (1) can also be written in the canonical form

$$S = -\frac{1}{2} \int d^4x \sqrt{-\bar{g}} \left(\bar{R} - \frac{1}{2}(\bar{\nabla}\phi)^2 \right) \tag{2}$$

where $\bar{g}_{\mu\nu} = \chi g_{\mu\nu}$ is the canonical metric and $\phi = \sqrt{2\omega + 3} \ln\chi$.

In this letter, we study the evolution of a flat [2] homogeneous isotropic universe with matter coupled minimally to $g_{\mu\nu}$ [1]-[5],[16]. The matter is taken to be a perfect fluid with density ρ (> 0) and pressure p, related by the equation of state $p = \gamma\rho$ where γ is a constant and $-1 \leq \gamma \leq 1$. [3] The fields depend on the time coordinate t only and the line element is given by $ds^2 = -dt^2 + e^{2A(t)} \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 \right)$ where e^A is the scale factor. Defining $\Omega \equiv 2\omega + 3$ and using $p = \gamma\rho$, the equations of motion become

$$\dot{A} = -\frac{\dot{\chi}}{2\chi} + \sqrt{\frac{\rho}{6\chi} + \frac{\Omega\dot{\chi}^2}{12\chi^2}} \tag{3}$$

$$\ddot{\chi} = -3\dot{A}\dot{\chi} + \frac{(1 - 3\gamma)\rho}{2\Omega} \tag{4}$$

$$\rho = \rho_0 e^{-3(1+\gamma)A} \tag{5}$$

where upper dots denote t-derivatives and ρ_0 is a positive constant. The second term in (3) is taken to be positive so that one obtains an expanding universe ($\dot{A} > 0$) of the standard cosmology when $\dot{\chi} = 0$ identically.

The analysis of the evolution is straightforward if equations (3)-(5) can be solved explicitly for all times with ρ and γ arbitrary. Since this is not possible, [4] we adapt a different method [21, 22]. Note that equation (4) can be integrated once to obtain

$$\dot{\chi}(t) = e^{-3A} \left(\sigma(t) + c \right), \quad \sigma(t) \equiv \frac{(1 - 3\gamma)\rho_0}{2\Omega} \int_{t_i}^{t} dt\, e^{-3\gamma A} \tag{6}$$

[2]It is straightforward to incorporate spatial curvature also.

[3]The range of γ includes the values corresponding to all known forms of matter such as, for example, vacuum energy density ($\gamma = -1$), dust ($\gamma = 0$), radiation ($\gamma = \frac{1}{3}$), and massless scalar fields dust ($\gamma = 1$).

[4]However, asymptotic power law solutions can be obtained [20].

and $c = \dot{\chi}e^{3A}(t_i)$ is a constant. Then, dividing equation (3) by $\frac{\dot{\chi}}{\chi}$ and substituting (6) for $\dot{\chi}$, we obtain an equation relating A and χ:

$$2\chi \frac{dA}{d\chi} = -1 + \text{sign}(\dot{\chi}) \sqrt{K} , \quad K \equiv \frac{\Omega}{3} \left(1 + \frac{2\rho_0 \chi e^{3(1-\gamma)A}}{(\sigma(t) + c)^2} \right) . \quad (7)$$

Also note that all the curvature invariants are finite and, hence, the singularities are absent if e^{-A} and $\frac{d^n A}{dt^n}$, $n \geq 1$ or, equivalently as follows from the repeated use of (3) - (5), if

$$e^{-A}, \ \frac{\rho}{\chi}, \ \text{and} \ \frac{\dot{\chi}}{\chi} \quad (8)$$

all remain finite [21, 22]. Using equations (5)-(7) and this sufficiency condition, the evolution can be analysed completely and the presence/absence of singularities determined even in the absence of explicit solutions.

3. We start at an initial time $t_{initial} \equiv 0$, corresponding to a temperature say $\gtrsim 10^{16}$ GeV. This is so that various model dependent phenomena such as Grand Unification symmetry breaking, inflation, etc., which are in any case not relevent to the issue of singularity, may all occur for $t > 0$ only. For initial conditions we choose $\dot{A}(0) > 0$, $\chi(0) > 0$, and $\dot{\chi}(0) > 0$ [5] corresponding to the fact that the universe is expanding and the string coupling is decreasing for $t \geq 0$. Their values, assumed to be non infinitesimal, are not required in the following. Also, $\chi(0)$ is assumed to be finite.

For $t > 0$, the scale factor e^A and the dilaton χ increase. In this era, various model dependent phenomena such as Grand Unification symmetry breaking, inflation, etc. may occur. However, they do not lead to singularity in any of the known models. Moreover, e^A and χ both continue to increase during and after these phenomena. Eventually $\chi \to \infty$ and $e^A \to \infty$ and, as $t \to \infty$, the asymptotic solution that must describe the present era is given by

$$e^A = e^{A_0} t^n , \quad \chi = \chi_0 t^m \quad (9)$$

where A_0 and χ_0 are constants and $(n, m) = (\frac{1}{2}, -\frac{1}{2})$ in the radiation dominated era $(\gamma = \frac{1}{3})$ and $= (\frac{2\omega+2}{3\omega+4}, \frac{2}{3\omega+4})$ in the dust dominated era $(\gamma = 0)$ [20].

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[5]If $\dot{\chi}(0) < 0$ one merely starts in Phase 4 (see below).

But this cannot be the entire story. Note that in string theory $\omega = -1$ but experimental observations require that $\omega > 500$. This contradiction is avoided if string theory generates a potential for χ and thereby, or otherwise, freezes its dynamics. Such a mechanism, however, is not expected to lead to any singularity. Therefore, in this letter, we assume the existence of such a mechanism and will not pursue it further.

3 a. Consider $t < 0$. Let $t' \equiv -t$ so that $t' > 0$ in this era. Then, in terms of t', the initial conditions are $\dot{A}(0) < 0$, $\chi(0) > 0$, and $\dot{\chi}(0) < 0$ where upper dots now denote t'-derivatives. Equation (5) remains unchanged whereas equations (3), (6), and (7) become

$$\dot{A} = -\frac{\dot{\chi}}{2\chi} - \sqrt{\frac{\rho}{6\chi} + \frac{\Omega\dot{\chi}^2}{12\chi^2}} \tag{10}$$

$$\dot{\chi}(t') = e^{-3A}\left(\sigma(t') + c\right) \tag{11}$$

$$2\chi\frac{dA}{d\chi} = -1 - \text{sign}(\dot{\chi})\sqrt{K} \tag{12}$$

where the constant $c = \dot{\chi}e^{3A}(0)$ is negative and

$$\sigma(t') = \frac{(1 - 3\gamma)\rho_0}{2\Omega}\int_0^{t'} dt' e^{-3\gamma A} \tag{13}$$

$$K = \frac{\Omega}{3}\left(1 + \frac{2\rho_0\chi e^{3(1-\gamma)A}}{(\sigma(t') + c)^2}\right). \tag{14}$$

It follows from the initial conditions that $\chi\frac{dA}{d\chi}(0) = \chi\frac{\dot{A}}{\dot{\chi}}(0) > 0$ and, hence from (12), that $K(0) > 1$.

Consider the evolution for $t' > 0$. Clearly, for $t' > 0$, the universe is dominated by radiation ($\gamma = \frac{1}{3}$) or, when e^A is sufficiently small, by massless scalar fields ($\gamma = 1$). Therefore, $(1 - 3\gamma) \leq 0$. As t' increases e^A decreases and, hence, the integral in (13) increases. This implies, since $(1 - 3\gamma) \leq 0$, that $\sigma(t')$ decreases or remains constant. Therefore, $(\sigma(t') + c) \leq c < 0$. Consequently, as t' increases, $\dot{\chi} < 0$ and, hence, χ decreases.

Since e^A and χ decrease, and $(\sigma(t') + c)^2$ increases or remains constant, it follows that K decreases monotonically. Its lowest value is $\frac{\Omega}{3} \leq \frac{1}{3}$, achievable when χ vanishes, see (14). Since $K(0) > 1$, it then follows that there exists a time, say $t' = t'_m > 0$ where $K(t'_m) = 1$ with $\chi(t'_m) > 0$. Therefore,

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$\frac{dA}{d\chi}(t'_m) = 0$ implying that $\dot{A}(t'_m) = 0$. This is a critical point of e^A and is a minimum. Also, equation (12) gives

$$A(t'_m) - A(0) = \int_{\chi(0)}^{\chi(t'_m)} \frac{d\chi}{2\chi} \left(-1 + \sqrt{K}\right) = \text{finite} , \qquad (15)$$

where the last equality follows because both the integrand and the interval of integration are finite. This implies that $A(t'_m)$ is finite and, therefore, that $e^{A(t'_m)}$ is finite and non vanishing.

It can be seen that the quantities in (8) are all finite implying that the curvature invariants are all finite. Thus, there is no singularity for $0 \le t' \le t'_m$.

3 b. Let $t'_1 \equiv t'_m + \delta$ where δ is a positive infinitesimal constant. Then, by continuity, we have $\dot{A}(t'_1) > 0$, $\dot{\chi}(t'_1) < 0$, $(\sigma(t'_1) + c) < 0$, and $K(t'_1) < 1$. Thus, for $t' > t'_1$, χ decreases and e^A increases. However, nothing can be said about $(\sigma(t') + c)$ or $K(t')$.

To proceed further, assume that $K(t') < 1$ for all $t' > t'_1$. This necessarily requires that $\chi e^{3(1-\gamma)A}$ remain finite and, since $(\sigma(t'_1) + c) < 0$, that $(\sigma(t') + c)$ remain negative and non infinitesimal. Therefore, $\dot{\chi} < 0$ and χ decreases continuosly whereas $\dot{A} > 0$ and e^A increases continuosly. Then, in the limit $\chi \to 0$, equations (10)-(12) can be solved explicitly. The solution is [20, 23, 22]

$$\text{for } \Omega \ne \frac{1}{3} : \quad e^A = e^{A_0} \left(t'_s - \text{sign}(m)t'\right)^n , \quad \chi = \chi_0 \left(t'_s - \text{sign}(m)t'\right)^m ,$$
$$(16)$$

where A_0, $\chi_0 > 0$, and $t'_s > t'_1$ are constants, and

$$n = \frac{3 - \sqrt{3\Omega}}{3(1 - \sqrt{3\Omega})} , \quad m = \frac{-2}{1 - \sqrt{3\Omega}} ; \qquad (17)$$

$$\text{for } \Omega = \frac{1}{3} : \quad e^A = e^{A_0} e^{kt'} , \quad \chi = \chi_0 e^{-3kt'} , \qquad (18)$$

where $k > 0$ is a constant.

If $\Omega > \frac{1}{3}$ then $m > 0$ and $n < 0$. Thus, as $\chi \to 0$, $t' \to t'_s$ and $e^A \to \infty$, and it can be seen that the curvature scalar diverges. Therefore, there is a singularity at a finite time t'_s.

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If $\Omega < \frac{1}{3}$ then $m < 0$ and $n > 0$. Thus, as $\chi \to 0$, $t' \to \infty$ and $e^A \to \infty$. It can then be seen that the quantities in (8) are all finite for $t'_1 \leq t' \leq \infty$, implying that all the curvature invariants are finite and, hence, there is no singularity. This is true for $\Omega = \frac{1}{3}$ also.

For strings, $\Omega = 1$ and therefore the solution is given by (16) with $n = -\frac{1}{\sqrt{3}}$ and $m = \sqrt{3} + 1$. Thus there is a singularity at a finite time t'_s. [6]

Is this singularity unavoidable? Equivalently, is $K(t') < 1$ for all $t' > t'_1$? Consider the case where $\Omega = 1$ and, as $e^A \to \infty$, the universe is dominated by 'matter' for which $\gamma \leq -\frac{1}{\sqrt{3}}$. [7] Then $(1 - 3\gamma) > 0$ and, as $t' \to t'_s$, it can be seen easily that $\sigma(t') \to \infty$ (logarithmically if $\gamma = -\frac{1}{\sqrt{3}}$). Also, $\chi e^{3(1-\gamma)A}(t') \to \infty$. This implies that $(\sigma(t') + c)$ which is negative initially at t'_1 must cross zero before t'_s and, hence, $K(t')$ must diverge. Since $K(t'_1) < 1$, it thus follows that there exists a time, say $t' = t'_M$ ($t'_1 < t'_M < t'_s$), where $K(t'_M) = 1$ and, hence, $\chi \frac{dA}{d\chi}(t'_M) = 0$. It also follows necessarily that $e^{A(t'_M)} < \infty$ (see (16)), $\chi(t'_M) > 0$ (by reversing the argument which led to (15)), $(\sigma(t'_M) + c) < 0$ (otherwise K diverges), and $\dot{\chi}(t'_M) \neq 0$ (see (11)). Therefore, it now follows that $\dot{A}(t'_M) = 0$. This is a critical point of e^A and is a maximum.

It can be seen that the quantities in (8) are all finite for $t'_1 \leq t' \leq t'_M$, implying that all the curvature invariants are finite. Thus, there is no singularity for $t'_1 \leq t' \leq t'_M$.

3 c. Let $t'_2 \equiv t'_M + \delta$. Then, by continuity, we have $\dot{A}(t'_2) < 0$, $\dot{\chi}(t'_2) < 0$, $(\sigma(t'_2) + c) < 0$, and $K(t'_2) > 1$. If $(\sigma(t'_2) + c)$ remains negative and does not vanish for $t' > t'_2$ then the initial conditions at t'_2 are same as those at $t' = 0$ and, hence, the evolution proceeds as in **3 a**. Thus, the evolution becomes cyclical but remains singularity free. The dilaton χ decreases continuosly but remains finite and non vanishing for $t' < \infty$.

If $(\sigma(t'_2) + c)$ vanishes at time, [8] say $t' = t'_n > t'_2$ then it implies that

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[6]This singularity is similar to the one encountered in the superinflationary branch of the pre big bang cosmology [4, 5] where it leads to the graceful exit problem which, perhaps, may also be solved by the resolution to be given below.

[7]More generally, by 'matter' for which $\gamma \leq \frac{1-\sqrt{3\Omega}}{3-\sqrt{3\Omega}}$ - thus, $\gamma \leq 0$ for $\Omega = \frac{1}{3}$ and $\gamma \leq \frac{1}{3}$ for $\Omega = 0$. When such 'matter' is present the evolution with $\Omega < 1$ also proceeds as described below, and not as given by equations (16)-(18).

[8]This is likely to be the case since, as clear from the evolution for $t' \leq t'_M$ in **3 b**, $(\sigma(t') + c)$ is increasing rapidly at t'_M.

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$\dot{\chi}(t'_n) = 0$. This is a critical point of χ and is a minimum.

It can be seen that the quantities in (8) are all finite for $t'_2 \leq t' \leq t'_n$, implying that all the curvature invariants are finite. Thus, there is no singularity for $t'_2 \leq t' \leq t'_n$.

3 d. Let $t'_3 \equiv t'_n + \delta$. Then, by continuity, we have $\dot{A}(t'_3) < 0$, $\dot{\chi}(t'_3) > 0$, and $(\sigma(t'_3) + c) > 0$. As t' increases, χ increases and e^A decreases and, in the absence of any other effects, e^A would vanish at time, say $t' = t'_z > t'_3$.

However, it follows from (5) that when e^A is sufficiently small the universe is dominated by massless scalar fields since $\gamma = 1$ for them. Then $(1-3\gamma) < 0$ and $\sigma(t')$, and hence $(\sigma(t')+c)$, begin to decrease. As $\chi \to \infty$, it follows that $\frac{\rho}{\chi}$-term dominates $\frac{\dot{\chi}}{\chi}$-terms in equation (10). This implies that e^A decreases faster than $(t'_z - t')^{\frac{1}{3}}$ as $t' \to t'_z$. Hence, $\sigma(t') \to -\infty$ faster than $\ln(t'_z - t')$ implying that $(\sigma(t') + c)$ which is positive initially at t'_3 must vanish at time, say $t' = t'_N$ $(t'_3 < t_N < t'_z)$. Consequently $\dot{\chi}(t'_N) = 0$. This is a critical point of χ and is a maximum.

It can be seen that the quantities in (8) are all finite for $t'_3 \leq t' \leq t'_N$, implying that all the curvature invariants are finite. Thus, there is no singularity for $t'_3 \leq t' \leq t'_N$.

3 e. Let $t'_4 \equiv t'_N + \delta$. Then, by continuity, we have $\dot{A}(t'_4) < 0$ and $\dot{\chi}(t'_4) < 0$. Also, $(\sigma(t'_4) + c) < 0$, and $K(t'_4) > 1$. Therefore, the initial conditions at t'_4 are same as those at $t' = 0$ and, hence, the evolution proceeds as described in **3 a**. Thus, the evolution becomes cyclical but remains singularity free. Also, during the course of the evolution, χ remains finite and non vanishing.

We now summarise the evolution (we use the original time variable $t \equiv -t'$ and upper dots now denote t-derivatives).

Phase 0 $(t \geq 0)$: In this phase, various model dependent phenomena such as Grand Unification symmetry breaking, inflation, generation of dilaton potential, etc. may occur but they do not lead to singularities. For $t \leq 0$ we have

Phase 1 $(-t_1 \leq t \leq 0)$: $\dot{A}(0) > 0$, $\dot{\chi}(0) > 0$. As t decreases, both e^A and χ decrease. Then radiation $(\gamma = \frac{1}{3})$ or massless scalar fields $(\gamma = 1)$ dominate the universe as e^A becomes small. Under these conditions, e^A always reaches a non zero minimum at time say $-t_1 + \delta$ where δ is a positive infinitesimal constant. For $t \leq -t_1$ it leads to

Phase 2 $(-t_2 \leq t \leq -t_1)$: $\dot{A}(-t_1) < 0$, $\dot{\chi}(-t_1) > 0$. As t decreases, e^A

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increases and $\to \infty$ and χ decreases and $\to 0$. If $\Omega \leq \frac{1}{3}$ then $e^A \to \infty$ and $\chi \to 0$ as $t \to -\infty$ and there is no singularity. If $\Omega = 1$ as in (1) then $e^A \to \infty$ and $\chi \to 0$ in a finite time and there is a singularity. However, in both cases, if there exists 'matter' for which $\gamma \leq \frac{1-\sqrt{3\Omega}}{3-\sqrt{3\Omega}}$ then it dominates the universe as e^A becomes large. Under these conditions, e^A always reaches a finite maximum at time say $-t_2 + \delta$, thereby avoiding the singularity in the case of $\Omega = 1$ (and, more generally, in the case of $\Omega > \frac{1}{3}$ also). For $t \leq -t_2$ it leads to

Phase 3 $(-t_3 \leq t \leq -t_2)$: $\dot{A}(-t_2) > 0$, $\dot{\chi}(-t_2) > 0$. As t decreases, e^A and χ continue to decrease. χ may or may not reach a non zero minimum. In the later case, Phase 3 is identical to Phase 1. In the former case, let χ reach a minimum at time say $-t_3 + \delta$. For $t \leq -t_3$ it leads to

Phase 4 $(-t_4 \leq t \leq -t_3)$: $\dot{A}(-t_3) > 0$, $\dot{\chi}(-t_3) < 0$. As t decreases, χ increases and e^A decreases. Then massless scalar fields ($\gamma = 1$) dominate the universe as e^A becomes small. Under these conditions, χ always reaches a finite maximum at time say $-t_4 + \delta$. For $t \leq -t_4$ it leads to

Phase 5 the initial conditions at the beginning of which are the same as those in Phase 1. Hence, the evolution also proceeds as described in Phase 1.

4. The evolution is thus completely singularity free under the conditions given in Phase 2 during which e^A increases, χ decreases and, hence, the effective string coupling $g_s = \frac{1}{\sqrt{\chi}}$ increases. During this Phase, if Ω becomes $\leq \frac{1}{3}$ and if no 'matter' exists for which $\gamma \leq \frac{1-\sqrt{3\Omega}}{3-\sqrt{3\Omega}}$ then $e^A \to \infty$ and $\chi \to 0$ as $t \to -\infty$, given by (16)-(18); if such 'matter' exists then, for any $\Omega \leq 1$, both e^A and χ evolve cyclically and remain finite and non vanishing as $t \to -\infty$.

It appears that either or both of these possibilities, namely Ω becomes $\leq \frac{1}{3}$ and/or 'matter' exists for which $\gamma \leq \frac{1-\sqrt{3\Omega}}{3-\sqrt{3\Omega}}$, can be realised in string theory by a gas of p-branes - the string solitons. To begin with, it is highly plausible that a gas of p-branes dominate the universe in Phase 2. Note that in string units, p-branes have masses $\sim \frac{1}{g_s}$ or $\frac{1}{g_s^2}$ [14, 15]. Hence, as g_s becomes large, p-branes become light and, hence, are likely to be produced copiously thus dominating the universe during Phase 2.

Duff *et al* have derived the metric $\tilde{g}_{\mu\nu}$ to which the (solitonic) p-branes couple minimally [16]. In terms of $\tilde{g}_{\mu\nu}$ and the p-brane dilaton $\tilde{\chi}$, related to

$g_{\mu\nu}$ and χ by

$$g_{\mu\nu} = \tilde{\chi}^{1 + \frac{p+1}{\sqrt{p^2+3}}} \tilde{g}_{\mu\nu} \ , \qquad \chi = \tilde{\chi}^{- \frac{p+1}{\sqrt{p^2+3}}} \ , \tag{19}$$

the graviton-dilaton action can be written as in (1) but now with a ω given by [9]

$$\tilde{\Omega} \equiv 2\tilde{\omega} + 3 = \frac{(p+1)^2}{p^2 + 3} \ . \tag{20}$$

Note that for $p = 0$ and -1, $\tilde{\Omega} = \frac{1}{3}$ and 0 respectively. The evolution can now be analysed as in **3 b** [10] and it follows that there is no singularity for these values of Ω. If there also exists 'matter' for which $\gamma \leq \frac{1 - \sqrt{3\tilde{\Omega}}}{3 - \sqrt{3\tilde{\Omega}}}$ then further evolution proceeds as described in **3 b - 3 e**: the scale factor and the dilaton evolve cyclically and both remain finite and non vanishing.

The heavy solitonic p-branes becoming light and produced copiously, thus resolving the singularity is reminiscent of the recently discovered field theory mechanism [17]-[19] where heavy states - monopoles in [17], Ramond-Ramond black holes in [18], Dirichlet instantons in [19] - become light and resolve the moduli space singularities. Also, the strong coupling regime of the original theory (χ small) corresponds to the weak coupling regime of the solitons [11] ($\tilde{\chi}$ large), see (19).

Alternately, when e^A is increasing, a gas of p-branes may have negative pressure, and hence negative γ. In fact, γ is shown to be $= -\frac{1}{3}$ for strings ($p = 1$) [1]-[4]. Tseytlin considers the case $-\frac{1}{\sqrt{3}} < \gamma < \frac{1}{\sqrt{3}}$ also [3], although the nature of the corresponding 'matter' is not clear. (See [24] also for another (higher dimensional) context where a sufficient negative pressure may arise due to Dirichlet branes.) In the present case, the value of γ for arbitrary p is not known, but it is plausible that $\gamma \leq -\frac{1}{\sqrt{3}}$ for some p (suggested to us by G. Veneziano). If true then again the singularity in Phase 2 can be avoided and the evolution becomes singularity free.

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[9]In a sense the change of ω is analogous to, but more involved than, changing γ from 0 to $\frac{1}{3}$ in standard cosmology when one goes from dust dominated to radiation dominated universe. As in the standard cosmology, here too the intermediate dynamics is easy to study but it is expedient to simply switch from one value of ω to other.

[10]For $p = -1$, we first set $p = -1 + \delta$ and, after performing the analysis, take the limit $\delta \to 0$.

[11]The effective p-brane coupling can be seen, by using the action (1) and by an analogy with the effective string coupling, to be given by $\tilde{g}_p = \frac{1}{\sqrt{\tilde{\chi}}}$.

Thus, the resulting scenario for the resolution of cosmological singularity is that as the string coupling becomes strong in Phase 2, solitonic p-branes are produced copiously and dominate the universe. They either change ω in (1) from -1 to $\tilde{\omega}$ given in (20) or produce a negative pressure corresponding to $\gamma \leq -\frac{1}{\sqrt{3}}$ or both. The singularity in Phase 2 is then avoided and further evolution is singularity free.

5. A few remarks are now in order. First, we have considered various metrics - the string σ-model metric $g_{\mu\nu}$, the canonical metric $\bar{g}_{\mu\nu}$, and the p-brane metric $\tilde{g}_{\mu\nu}$ which are related to each other through dilaton dependent conformal factors (see (2) and (19)). Therefore when one metric is singularity free then the other two are also singularity free if the dilaton remains finite and non vanishing, which is ensured if 'matter' with appropriate γ exists (see Phase 2). If p-branes, $p = 0$ and/or -1, dominate the universe during Phase 2 but 'matter' with appropriate γ does not exist then p-brane metric $\tilde{g}_{\mu\nu}$ evolves as in (16) or (18) and is singularity free. It can be shown, following the methods of [21], that any metric $\hat{g}_{\mu\nu} \equiv \tilde{\chi}^{\alpha} \tilde{g}_{\mu\nu}$ is singularity free and the corresponding time can be continued indefinitely into the past and the future if and only if $\alpha \leq 1 - \sqrt{3\tilde{\Omega}}$. It therefore follows from (20) and (19) that when 0-branes dominate the universe $\tilde{g}_{\mu\nu}$ is singularity free but $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ are singular whereas when (-1)-branes dominate the universe all of these metrics are singularity free.

Second, action (1) is unlikely to be valid in the strong coupling regime. This is just as well since otherwise the evolution is singular and the singularities cannot be avoided. However, an effective action valid at strong coupling is not known. In a sense the above scenario, which ensures singularity free evolution of the universe, can be viewed as a conjecture towards obtaining such an action. Or, given the analogy between the above scenario and the resolution of moduli space singularities, the methods of [17]-[19] can perhaps be applied to understand the details of strong coupling effects near the cosmological singularities also.

Third, action (1) may receive higher derivative corrections, which must be included. However, when the curvature invariants all remain finite they may not be crucial to the cosmological evolution. Also, in string theory such corrections appear to ameliorate the singularity problem [7]-[9], so it is possible that they improve the singularity aspects in the present case also. However, further study is required to understand the effects, crucial or otherwise, of

higher derivative terms on cosmological evolution. Important though such a study is, we defer it to future as it is beyond the scope of the present work.

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